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14. ABSTRACT
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Resource Allocation in MIMO-based Ad Hoc Networks

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***Abstract*—Multiple-Input Multiple-Output (MIMO) communications have shown great promise in providing high spectral efficiency for wireless ad hoc networks. In this paper we study the problem of joint routing, scheduling, power control and bit rate selection in MIMO-based ad hoc network with the goal of maximizing the system throughput that satisfies a given end-to-end traffic demand. We formulate this cross-layer optimization problem as a linear programming problem. It is well known that the scheduling problem arising in wireless network is NP-hard due to the exponential optimization space. Accommodating MIMO links in this work greatly increases the computation complexity. The cross-layer optimization problem can be solved by applying an iterative approach proposed in [10], and the original problem is decomposed into subproblems and solve them iteratively. Numerical results are given to validate the solution.**

I. Introduction

Multiple-Input Multiple-Output (MIMO) systems have shown great promise in providing high spectral efficiency for single user wireless link without interference [1], [2]. There has also much work on the MIMO-based cellular networks, which include MIMO multiple access (MIMO-MAC) [3], [4] and MIMO broadcast systems [5], [6]. Both systems have one common end of the communication link – either the transmitter of MIMO-BS or the receiver of MIMO-MAC. There has been great interest in extending the MIMO communication to the multi-user systems with interference. The transmission scheme of each user depends on that of other users since the interferences at each user depend on all the transmit covariance matrices in the network.

A Gradient Projection (GP) algorithm [7] and a Quasi-Newton (QN) method [8] were presented to provide suboptimal solutions subject to the constant power constraint. In [9], we propose an Integer Programming (IP) method to study the total system capacity problem for the MIMO system with interference. Both transmitters and receivers are assumed to have perfect knowledge of channel state information (CSI). Each MIMO link is decomposed into parallel independent eigenchannel. Then, each eigenchannel is represented by a set of logical links, and each logical link is determined by different discrete powers and discrete data rates. Also, the transmitters of each MIMO link are subject to the total power constraint. The solution specifies the set of logical links that can transmit simultaneously.

The broadcast nature of wireless radio causes interference among the nodes sharing the common communication media. It is well known that the optimal scheduling problem in wireless network is NP-hard. The optimization space grows exponentially with the number of links. For example, if single channel is used, then one must maximize over a polytope with 2^L extreme points, where L is the number of links. More specifically, we define an *assignment* to be a specification of which links are transmitting and which links are not transmitting. A *schedule* is the convex sum of assignments. The main challenge to the throughput maximization problem has been that the space of all assignments is too large. In order to solve the scheduling problem, the space of assignments must be reduced. In [10], the authors have developed the iterative techniques to compute the optimal scheduling for the wireless mesh network even when co-channel interference is present.

Allowing the schedule optimization to optimize over MIMO links results in a higher dimension problem. However, the main results and the algorithm for optimal scheduling from [10] are still available. In this paper, we study the problem of resource allocation in MIMO-based Ad Hoc networks by jointly determining routing, scheduling, power control and bit rate selection with the goal of maximizing the system throughput that satisfies an end-to-end traffic demand. After solving the optimization problem, the assignment specifies which links/eigenchannels are transmitting, their transmit powers and bit rates. Routing is also solved as a by-product. During searching for a new assignment, the dimension of the integer programming problem significantly increases by considering MIMO links.

The remainder of this work is organized as follows. Section II briefly introduces some related work. In Section III, an integer programming based algorithm [9] is presented to maximize the total system capacity for MIMO links with interference. In Section IV, problem definition are given, and an iterative approach to solve the resource allocation problem is presented. The approach focuses on reducing the size of the space of assignments and finding a new assignment. Then, Section V shows the results from numerical experiments. Finally, concluding remarks are given in Section VI.

The notations in this paper are as follows. The boldface denotes matrices and vectors. For a matrix $\mathbf{A}$, $\mathbf{A}^\dagger$ denotes the conjugate-transpose. $|\mathbf{A}|$ denotes the determinant, and $Tr(\mathbf{A})$ denotes the trace of a square matrix. $\mathbf{A}\succeq 0$ means that $\mathbf{A}$ is a positive semi-definite matrix. $\mathbf{A}$ is a Hermitian matrix if $\mathbf{A} = \mathbf{A}^\dagger$. $\mathbf{I}$ denotes the identity matrix with the appropriate dimension from the context. $E[\cdot]$ denotes the statistical expectation. $\mathbf{C}(x \times y)$ denotes the complex space of $x \times y$ matrix. $min(x, y)$ and $max(x, y)$ denotes the minimum and the maximum of two real numbers x and y, respectively.

II. Related Work

There is increasing interest in the MIMO-based ad-hoc network. In [11], Blum investigates the MIMO capacity with interference where single-user detection is assumed at the receiver. Without CSI at the transmitter, Blum shows that the optimum is either the optimum interference-free approach, which puts equal power into each antenna, or a singular mode, which puts all powers into a single antenna, if the interference is either sufficiently weak or sufficiently strong.

In [7], Ye and Blum study the asymptotic behavior of the system mutual information and propose a Gradient Projection (GP) algorithm to maximize the total system capacity subject to the constant power constraint. In [8], a Quasi-Newton (QN) method is proposed to solve the capacity problem by approximating the inverse of the Hessian matrix instead of computing the real one. However, only the suboptimal solutions can be found due to the non-convex nature of the optimization problem.

Utility optimization of wired networks was pioneered by the seminal works of Kelly [12], Low [13]. The same framework has been extended to ad hoc networks in [14], [15]. The main obstacle facing throughput maximization is that the size over which the optimization is performed is exponential with the number of links [16]–[18]. To reduce the optimization space, one approach is to arbitrarily restrict the focus to a subset of all assignments, which is followed in [18]. However, this approach reduces the resulting throughput. Another approach is to develop a heuristic to determine the subset of considered assignments [19], [20]. The performance of these methods typically decreases as the number of links increases [19], [20]. Finally, one can take a brute force approach, and consider the entire set of assignments. This approach is taken in [16], where, due to computational difficulties, the largest network that could be solved has only 15 links. However, the authors in [10] has developed the iterative techniques to compute the optimal scheduling for the wireless mesh network with more than 2,000 links.

III. Capacity of MIMO Links with Interference

Without considering routing and scheduling for end-to-end flows, we first consider the capacity of a MIMO interference system with L links where each node is equipped with n_t transmit antennas and n_r receive antennas. Suppose all MIMO nodes communicate in the same channel, and cochannel interference is presented when more than one link transmit simultaneously. The details of this section can be found in [9]. Note that we discuss random channel matrix in this section, but we only consider deterministic channel matrix when we include routing and scheduling.

A. MIMO Links with Interferences

We use $\mathbf{H}_{i,j} \in \mathbf{C}(n_r \times n_t)$ to represent the channel gain matrix from the transmit antennas of link j to the receive antennas of link i. The complex base-band signal vector received by the receiver node of link i is given by

$$\mathbf{y}_i = \sqrt{\rho_i}\mathbf{H}_{i,i}\mathbf{x}_i + \sum_{j=1,j\neq i}^{L} \sqrt{\rho_{i,j}}\mathbf{H}_{i,j}\mathbf{x}_j + \mathbf{n}_i,$$

where $\mathbf{x}_i$ represents the normalized transmitted signal of link i, ρ_i denotes the signal-to-noise ratio (SNR) of link i, $\rho_{i,j}$ denotes the interference-to-noise ratio (INR) of link i due to the interference from link j, and $\mathbf{n}_i$ is the noise vector. The entries of $\mathbf{H}_{i,j}$ and $\mathbf{n}_i$ are independent and identically distributed complex Gaussian random variables with zero mean and unit variance.

Let $\mathbf{Q}_i = E[\mathbf{x}_i\mathbf{x}_i^\dagger]$ be the covariance matrix for the transmit signal vector $\mathbf{x}_i$. $Tr(\mathbf{Q}_i)$ specifies the transmit powers of transmit antennas of link i, and we have $\mathbf{Q}_i \succeq 0$ and $Tr(\mathbf{Q}_i) \leq 1$ where 1 represents the normalized maximum power. The interference-plus-noise of link i is $\sum_{j=1,j\neq i}^{L} \sqrt{\rho_{i,j}}\mathbf{H}_{i,j}\mathbf{x}_j + \mathbf{n}_i$, which is Gaussian distributed with covariance matrix $\mathbf{R}_i = \mathbf{I} + \sum_{j=1,j\neq i}^{L} \rho_{i,j}\mathbf{H}_{i,j}\mathbf{Q}_j\mathbf{H}_{i,j}^\dagger$ where $\mathbf{I} = \mathbf{E}(\mathbf{n}_i\mathbf{n}_i^\dagger)$. The ergodic capacity [2] of link i can be written as

$$C_i = E\left(\log_2 \left|\mathbf{I} + \rho_i\mathbf{H}_{i,i}\mathbf{Q}_i\mathbf{H}_{i,i}^\dagger\mathbf{R}_i^{-1}\right|\right)$$

where the expectation is taken over the distribution of $\mathbf{H}$.

The optimization problem to maximize the system capacity can be defined as

$$\max \sum_{i=1}^{L} w_i C_i \tag{1a}$$
$$\text{subject to} \quad Tr(\mathbf{Q}_i) \leq 1$$
$$\mathbf{Q}_i \succeq 0.$$

where w_i are pre-assigned weights. Due to the nonconvexity of this problem, only a suboptimal solution can be found.

B. Capacity of MIMO Links via SVD

Assume the MIMO interference system with a rich scattering environment. Singular Value Decomposition (SVD) can be used to obtain the independent channels for a MIMO link. Consider a MIMO link with $n_r \times n_t$ channel gain matrix $\mathbf{H}$ that is known to both the transmitter and the receiver. The channel gain matrix $\mathbf{H}$ is decomposed into

$$\mathbf{H} = \mathbf{U}\mathbf{\Lambda}\mathbf{V}^\dagger,$$

where the $n_r \times n_r$ matrix $\mathbf{U}$ and the $n_t \times n_t$ matrix $\mathbf{V}$ are unitary matrices, and the $n_r \times n_t$ matrix $\mathbf{\Lambda}$ is a diagonal matrix of singular values $\{\sigma_i\}$ of $\mathbf{H}$. Due to considering a rich scattering environment, the matrix $\mathbf{H}$ is full rank and $Rank(\mathbf{H}) = M = \min(n_r, n_t)$.

The channel gain matrix $\tilde{\mathbf{H}}_{ij}$ from the transmitters of eigenchannels of link j to the receivers of eigenchannels of link i $(i \neq j)$ shown in Fig 1 can be written as

$$\tilde{\mathbf{H}}_{i,j} = \mathbf{U}_i^\dagger\mathbf{H}_{i,j}\mathbf{V}_j, \tag{2}$$

where $\mathbf{U}_i$ and $\mathbf{V}_j$ are from the SVD of matrices $\mathbf{H}_{i,i}$ and $\mathbf{H}_{j,j}$. Then, $\tilde{H}_{i,j}^{l,k}$ denotes the channel gain from the

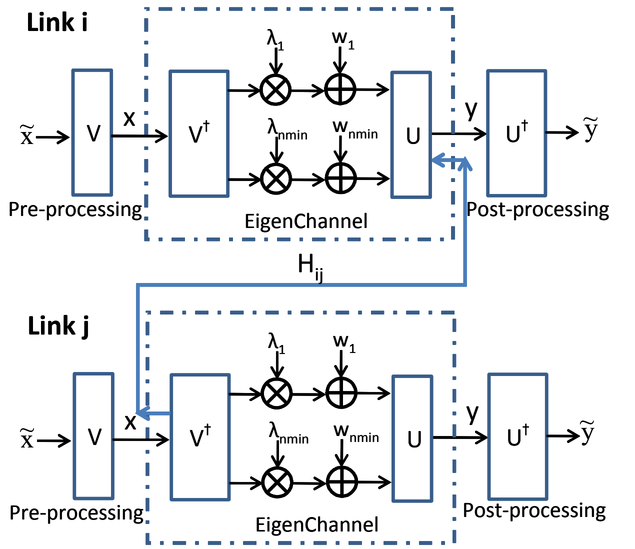

Fig. 1. The channel gain matrix $\tilde{\mathbf{H}}_{ij}$ from the transmitters of eigenchannels of link j to the receivers of eigenchannels of link i

transmitter of the k^{th} eigenchannel of link j to the receiver of the l^{th} eigenchannel of link i.

The aggregate interference-plus-noise at the receiver of the l^{th} eigenchannel of link i has the covariance ζ_{i_l} written as

$$\zeta_{i_l} \approx 1 + \sum_{j=1, j \neq i}^{L} \rho_{ij} \sum_{k=1}^{M} P_{j_k} \tilde{H}_{i,j}^{l,k} \left(\tilde{H}_{i,j}^{l,k} \right)^{\dagger}.$$

where P_{j_k} is the transmit power at the k^{th} eigenchannel of link j. Then, the approximately ergodic capacity of link i can be written as

$$C_i \approx E\left(\sum_{l=1}^{M} \log_2 \left(1 + \frac{\rho_i \sigma_{i_l}^2 P_{i_l}}{\zeta_{i_l}} \right) \right) \tag{3}$$

where the expectation is taken over the distribution of $\mathbf{H}$, σ_{i_l} is the singular value of the l^{th} eigenchannel of link i, and P_{i_l} is the transmit power at the l^{th} eigenchannel of link i. The details can be found in [9]. From the proof, we know that equation (3) is very accurate when L and M are large.

In summary, the channel covariance coefficient D_{x_m, y_n} from the transmitter of the n^{th} eigenchannel of link y to the receiver of the m^{th} eigenchannel of link x can be represented as

$$D_{x_m, y_n} = \begin{cases} \sigma_{x_m}^2 & \text{if } x = y \text{ and } m = n \\ 0 & \text{if } x = y \text{ and } m \neq n \\ \tilde{H}_{x,y}^{m,n} \left(\tilde{H}_{x,y}^{m,n} \right)^{\dagger} & \text{if } x \neq y. \end{cases}$$

C. Integer Programming Based Algorithm

Suppose that there are K modulation/coding schemes and S transmit powers available for the m^{th} eigenchannel of link x, then associated with each MIMO link x is the set of logical links $x_{m,k,s}$ with $1 \leq m \leq M$, $1 \leq k \leq K$ and $1 \leq s \leq S$. We define an assignment $\mathbf{v}$ as a set of logical links that can transmit simultaneously. Specifically, an assignment $\mathbf{v} \in \{0,1\}^{L \times M \times K \times S}$, where $v_{x_{m,k,s}} = 1$ implies that the MIMO link x is transmitting at the m^{th} eigenchannel, the k^{th} bit-rate and the s^{th} normalized power.

We define γ as the buffer size to reduce the sensitivity to interference. For a particular ρ_x, the maximum data rate of the m^{th} eigenchannel of link x is

$$C_{x_m}^{\max} = \log_2 \left(1 + \frac{\rho_x \sigma_{x_m}^2 P_{\max}}{\gamma} \right),$$

where $P_{\max} = 1$ is the normalized maximum power and $\gamma = 2dB$ is used in this work.

For simplicity, we select K linearly separated data rates denoted by $[C_{x_m}^{\max}/K \ ... \ C_{x_m}^{\max}]$. Then, the $SINR$ threshold to achieve the k^{th} data rate over the m^{th} eigenchannel of link x can be represented as

$$SINR_{x_{m,k}}^{th} = 2^{k \times C_{x_m}^{\max}/K} - 1.$$

Note that $SINR_{x_{m,k}}^{th}$ depends on the link x. Similarly, the set of powers can be selected as S linearly separated transmit powers denoted by $[1/S \ ... \ 1]$.

Let $R_{x_{m,k,s}}$ be the data rate over the logical link $x_{m,k,s}$. The aggregate interference-plus-noise at the receiver of the m^{th} eigenchannel of link x has the covariance ζ_{x_m} which can be written as

$$\zeta_{x_m} = \sum_{y=1, y \neq x}^{L} \rho_{x,y} \sum_{n=1}^{M} D_{x_m, y_n} \sum_{t=1}^{K} \sum_{q=1}^{S} P_{y_{n,t,q}} v_{y_{n,t,q}} + 1,$$

where $P_{y_{n,t,q}}$ is the transmit power of logical link $y_{n,t,q}$. The logical link $x_{m,k,s}$ to be active must satisfy

$$\frac{\rho_x D_{x_m, x_m} P_{x_{m,k,s}}}{\zeta_{x_m}} > SINR_{x_{m,k}}^{th},$$

which is equivalent to

$$\rho_x D_{x_m, x_m} P_{x_{m,k,s}} - SINR_{x_{m,k}}^{th} \zeta_{x_m} > \infty \times \left(v_{x_{m,k,s}} - 1 \right). \tag{4}$$

In practice, ∞ is replaced with some large number. Define $O(n)$ and $I(n)$ as the sets of links that are outgoing from and incoming to node n. For simplicity, we assume that only one link can transmit if multiple links share one common node.

Problem (1) can be approximated by solving the following integer programming problem.

$$\max_{\mathbf{v}} \sum_{x=1}^{L} w_x \sum_{m=1}^{M} \sum_{k=1}^{K} \sum_{s=1}^{S} R_{x_{m,k,s}} v_{x_{m,k,s}}$$

subject to

$$\rho_x D_{x_m, x_m} P_{x_{m,k,s}} - SINR_{x_{m,k}}^{th} \zeta_{x_m} > \Gamma_{x_{m,k,s}} \left(v_{x_{m,k,s}} - 1 \right)$$

$$\text{for each logical link } x_{m,k,s} \tag{5a}$$

$$\sum_{m=1}^{M} \sum_{k=1}^{K} \sum_{s=1}^{S} P_{x_{m,k,s}} v_{x_{m,k,s}} \leq 1 \text{ for each link } x \tag{5b}$$

$$\sum_{\{x \in O(n); x \in I(n)\}} v_x \leq 1 \text{ for each node } n \tag{5c}$$

$$\sum_{k=1}^{K} \sum_{s=1}^{S} v_{x_{m,k,s}} \leq v_x \text{ for each eigenchannel of link } x$$

$$\tag{5d}$$

$$v_{x_{m,k,s}} \in \{0,1\},$$

where

$$\Gamma_{x_{m,k,s}} = SINR^{th}_{x_{m,k}} \times \left(\sum_{y=1, y \neq x}^{L} \rho_{x,y} \sum_{n=1}^{M} D_{x_m, y_n} \sum_{t=1}^{K} \sum_{q=1}^{S} P_{y_{n,t,q}} + 1 \right).$$

Constraint (5b) shows that the total power at the transmitters of link x must be no more than the normalized power 1. Constraint (5c) enforces that only one link can be active when multiple links share one common node n. Constraint (5d) enforces that no logical link of each eigenchannel of link x can be active if link x itself is inactive. One shortcoming of the IP algorithm is the computation difficulty, and some approximation algorithms based on Maximum Weighted Independent Set (MWIS) will be developed in the future research.

IV. Resource Allocation

A. Scheduling

We define an *assignment* $\mathbf{v}$ as a set of logical links that can transmit simultaneously. In other words, the assignment specifies which links are transmitting, which eigenchannels are active, their bit-rates, and their transmit powers. Specifically, an assignment $\mathbf{v} \in \{0,1\}^{L \times M \times K \times S}$, where $v_{x_{m,k,s}} = 1$ implies that the MIMO link x is transmitting at the m^{th} eigenchannel, the k^{th} bit-rate and the s^{th} normalized power. The *set of considered assignments* is denoted by $\mathcal{V}$, while the *set of all assignments* is denoted by $\overline{\mathcal{V}}$. The size of $\overline{\mathcal{V}}$ is the main challenge facing optimal scheduling. Thus, typically, we only consider a subset of all assignments, i.e., $\mathcal{V} \subsetneqq \overline{\mathcal{V}}$. We drop the terms physical and logical unless it is unclear from context.

A schedule is defined as a convex combination of assignments and can be represented by a set $\{\alpha_{\mathbf{v}}\}$ where $\alpha_{\mathbf{v}} \geq 0$ and $\sum_{\mathbf{v}} \alpha_{\mathbf{v}} \leq 1$ where $\alpha_{\mathbf{v}}$ is the fraction of time that the assignment $\mathbf{v}$ is used. Letting $R(\mathbf{v}, x_m)$ be the data rate over the m^{th} eigenchannel of link x during assignment $\mathbf{v}$, the average data rate over link x provided by schedule $\{\alpha_{\mathbf{v}}\}$ is $\sum_{\mathbf{v}} \alpha_{\mathbf{v}} \sum_m R(\mathbf{v}, x_m)$.

B. Routing

In this paper, the routing of MIMO-based ad hoc networks is studied in the multicommodity flow model. Let Φ be a set of concurrent sessions, and each session ϕ represents a source-destination pair in the network. A traffic demand d_ϕ for session ϕ has to be transmitted from the source node ϕ_s to the destination node ϕ_d via a collection of paths. That is, the flows transmit between the source-destination pairs through multi-path routing. All links are directed; bidirectional communication between two nodes is represented by two directed links.

We denote the data rate associated with the ϕ^{th} flow on link x by r_x^ϕ, and define $O(n)$ and $I(n)$ as the sets of links that are outgoing from and incoming to node n. At each node n, which is the source of flow ϕ, the following flow conservation law is satisfied:

$$\sum_{x \in O(n)} r_x^\phi = d_\phi.$$

If the node n is an intermediate relay node for the ϕ^{th} flow, the following balance equation holds:

$$\sum_{x \in O(n)} r_x^\phi - \sum_{x \in I(n)} r_x^\phi = 0.$$

If the node n is the destination for the ϕ^{th} flow, we have

$$-\sum_{x \in I(n)} r_x^\phi = -d_\phi.$$

Algorithm 1 Computing an Optimal Solution

1: Select an initial set of assignments $\mathcal{V}(0)$, set $i = 0$.
2: Solve (6) for $\mathcal{V} = \mathcal{V}(i)$ and compute $\boldsymbol{\mu}(i)$ and $\lambda(i)$, the Lagrange multipliers associated with constraints (6a) and (6b), respectively.
3: Search for an assignment $\mathbf{v}^+$ that solves (5)
4: **if** the assignment $\mathbf{v}^+ \in \mathcal{V}(i)$ or $\sum_{x=1}^{L} \mu_x(i) \sum_{m=1}^{M} R(\mathbf{v}^+, x_k) \leq \lambda(i)$ **then**
5: stop, the optimal schedule has been found.
6: **else**
7: set $\mathcal{V}(i+1) = \mathcal{V}(i) \cup \mathbf{v}^+$, set $i = i+1$, and go to Step 2.
8: **end if**

C. Resource Allocation Algorithm

Suppose the MIMO-based Ad Hoc network with L links and each node is equipped with M antennas. With this notation, the optimization problem of resource allocation can be described as

$$\max \beta$$

subject to:

$$\sum_{x \in O(n)} r_x^\phi = \beta d_\phi, \qquad n\text{: source of } \phi$$

$$\sum_{x \in O(n)} r_x^\phi - \sum_{x \in I(n)} r_x^\phi = 0, \qquad n\text{: relay of } \phi$$

$$-\sum_{x \in I(n)} r_x^\phi = -\beta d_\phi, \qquad n\text{: destination of } \phi$$

$$\sum_{\{\phi \in \Phi\}} r_x^\phi \leq \sum_{\mathbf{v} \in \mathcal{V}} \alpha_{\mathbf{v}} \sum_{m=1}^{M} R(\mathbf{v}, x_m) \text{ for each link } x \tag{6a}$$

$$\sum_{\mathbf{v} \in \mathcal{V}} \alpha_{\mathbf{v}} \leq 1 \tag{6b}$$

$$0 \leq \alpha_{\mathbf{v}} \text{ for each } \mathbf{v} \in \mathcal{V}. \tag{6c}$$

(6) is a linear programming problem, which is extensively studied in [21].

In theory, (6) is solvable. However, there is a significant computational challenge in that if $\mathcal{V}$ is the set of all assignments, then the vector $\boldsymbol{\alpha}$ has 2^{LMKS} elements. Thus, the size of the space over which the optimization is performed must be reduced. This idea of considering a reduced space was considered in [18] and [19], however, the space was constructed arbitrarily. In this work the space is constructed so that the throughput found by optimizing over the reduced space is the same throughput found by optimizing over the entire space.

The idea of Algorithm 1 is as follows. Given an initial set of randomly selected assignments $\mathcal{V}$, solve the throughput optimization problem over the set $\mathcal{V}$ and obtain the Lagrange multiplier for each link. Then, find a better assignment $\mathbf{v}^+$ by solving problem (5). If $\mathbf{v}^+$ exists, add it to the set $\mathcal{V}$ and go back to solve the optimization problem again. Otherwise, the optimal solution is obtained. The optimality and convergence property are proved in [10].

Because problem (6) over a small set of assignments is a linear programming problem, the computation complexity of Algorithm 1 is determined by finding a new assignment which is an integer programming problem. Specifically, each MIMO link is represented by a set of logical links. The dimension of problem (5) significantly increases with the number of MIMO links. Therefore, Algorithm 1 is fit for moderate size of MIMO-based Ad Hoc networks.

V. Numerical Experiments

In this section, we present some numerical results to investigate the resource allocation in MIMO-based Ad Hoc networks. We consider a randomly generated topology with 10 nodes uniformly distributed in a square region of 1500m x 1500m and the minimum distance between any two nodes is greater than 300m. Each node is equipped with two antennas and the maximum transmit power for each node is Pmax=0.1 watts (20 dBm). The numbers of data rates and powers for each eigenchannel are 8 and 4, respectively. Path loss is represented by the popular two-ray ground reflection model. The total bandwidth in the network is 20 MHz. There are two sessions in the network: node1 to node 2, and node 9 to node 7. For simplicity, the traffic demands for these sessions are [1 1] Mbps. The computations below were performed on a 2.4MHz AMD FX-53 processor with 8GB RAM, and CPLEX v10 is used to solve the LP and ILP problems.

A. Number of Iterations until Algorithm 1 Stops

Figure 2 shows how the throughput increases as the more assignments are added. The point of maximum throughput occurs when the stopping condition specified in Algorithm 1 is met. Thus, in this case, Algorithm 1 stopped after 43 iterations.

Note that only one assignment is added at each iteration. Thus, the maximum number of elements in $\mathcal{V}$ is the number of assignments in the initial set of assignments plus the number of iterations required by Algorithm 1. Hence, we have achieved the goal of determining the solution to (6) for $\mathcal{V} = \bar{\mathcal{V}}$ by computing the solution to (6) for a small set $\mathcal{V}$. The complexity of solving linear and nonlinear optimization problems is well known, and is not investigated here.

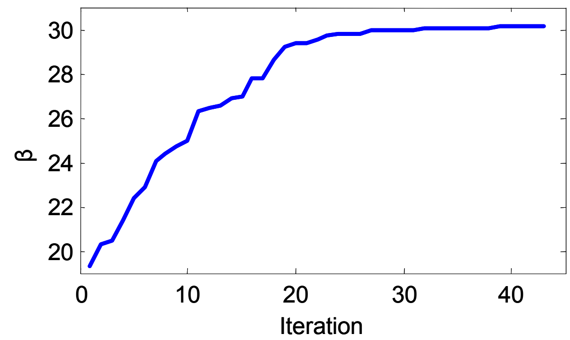

Fig. 2. Variation in the computed throughput as assignments are added.

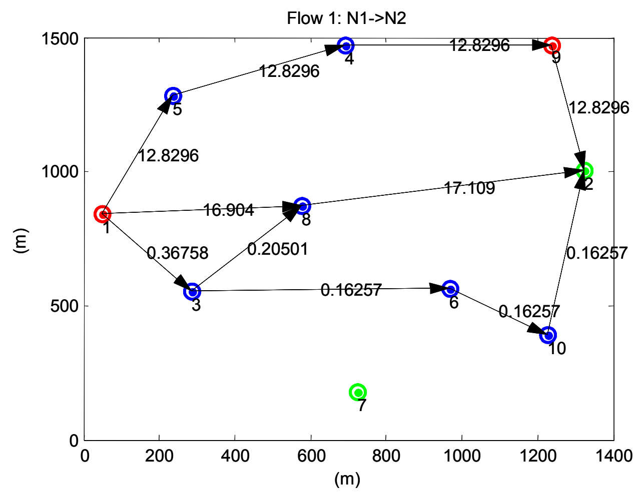

Fig. 3. Routes and flow rates for Session 1.

B. Optimal Routes and Flow Rates

Session 1 transmits from the leftmost node to the rightmost node, and Session 2 transmits from the topmost node to the lowest node. The routes and flow rates of session 1 and 2 are shown in Fig. 3 and Fig. 4, respectively. For both sessions, the optimal routes are multi-path multi-hop, and the flow rates satisfy the flow conservation constraints. Also, it is easy to see that bidirectional communication between two nodes is represented by two directed links.

VI. Conclusion

In this paper, we study the problem of resource allocation in MIMO-based Ad Hoc networks by jointly determining routing, scheduling, power control and bit rate selection with the goal of maximizing the system throughput that satisfies the end-to-end traffic demand. An iterative algorithm proposed in [10] was extended to solve the optimization problem of resource allocation. An Integer Programming based algorithm [9] is used to find a new assignment during each iteration, which specifies which links/eigenchannels are transmitting and their transmit powers and bit rates. After solving the optimization problem, joint routing, scheduling, power control and bit rates in MIMO-based Ad Hoc networks are founded and the optimal solution is obtained.

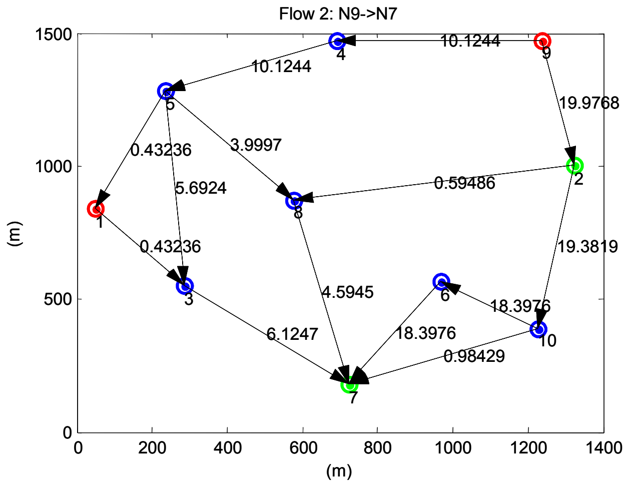

Fig. 4. Routes and flow rates for Session 2.

The computation complexity of the Integer Programming method prevents the proposed algorithm working for large size MIMO networks. In the future work, some greedy approximation methods will be developped to trade off the achieved performance and the computation complexity.

DISCLAIMER

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The views and conclusions contained in this document are those of the authors and should not be interpreted as representing the official policies, either expressed or implied, of the Air Force Research Laboratory or the U. S. Government.

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